CHAPTER 3

NEUTRON PHYSICS

The most distinct processes in a nuclear reactor are those involving neutrons. Neutrons initiate and maintain the fission chain reaction. Their energy, concentration, and multiplication determine the nuclear reaction rates and power densities. We will consider some of the introductory aspects of these processes beginning with a study of the interactions between neutrons and nuclei.

3.1 NEUTRON-NUCLEUS INTERACTIONS

Consider an ensemble of neutrons migrating in a medium which consists of one atomic specie identified by $A\chi$. The atoms may be viewed as being essentially stationary although the neutrons are always in motion with respect to the position of the atoms. Since the neutrons possess no net electric charge, their path of motion is unaffected by the negative electric field attributable to the electron cloud which surrounds each nucleus nor are they influenced by the positive electric field of the protons in the nucleus. The neutron can therefore interact most readily with the nucleus in any one of a variety of ways. Some of the more obvious and important interactions can be descirbed as follows:

1. Neutron capture: the nucleus captures a neutron and becomes transmuted into a different or unstable isotope of the same element.
2. Elastic scattering: the neutron undergoes a billiard-ball type collision with a nucleus.
3. Nuclear Fission: following addition of a neutron to the nucleus, the nucleus breaks up into two fission fragments.

We consider a discussion of the above neutron-nucleus interactions with an emphasis on those processes which are of particular importance in a CANDU reactor.

The capture process represents a transmutation whereby the nucleus becomes transformed into another stable or unstable isotope of the same element. For example, the deuterium nucleus in the moderator has a small affinity to capture neutrons. This may be represented by

$$D + n \rightarrow T \ . \qquad (3.1)$$

Here D represents a deuterium nucleus (Hydrogen-2) and T is a tritium nucleus (Hydrogen-3). Tritium is a radioactive beta emitter and decays to stable Helium-3 by beta decay

$$T \rightarrow {}^{3}He + {}^{-}\beta, \qquad (3.2)$$

with a half-life of 12.3 years; the beta particle has an energy of 18 keV. This process if of considerable regulatory importance because tritium, being a hydrogen isotope, combines with oxygen to form water and therefore could be ingested.

Many structural elements in a reactor contain trace elements of cobalt which exists as Cobalt-59 with 100% abundance. Neutron capture leads to unstable Cobalt-60 which decays to Nickel-60 with a half-life of 5.26 years and a metastable state with $T_{1/2}$ = 10.5 minutes. Several excited states in nickel will be attained each of which eventually decays to the ground state. Symbolically we may write this process as

$$^{59}Co + n \rightarrow {}^{60}Co \rightarrow {}^{60}Ni + \beta^{-} \quad . \qquad (3.3)$$

One undesirable feature of this process is that radioactive Cobalt-60 may, under some conditions, become transported through the primary cooling system and thus present a radiation hazard. However, because Cobalt-60 is a useful isotope in industrial radiography and medical therapy, it may be produced purposely in a power reactor. The detailed energy level structure of the Cobalt-60 decay to excited levels of Nickel-60 are illustrated in Fig. 3.1. Note the energies of the gamma rays associated with this decay.

As a final example of neutron absorption we cite here neutron capture in Boron-10 which leads to the emission of an alpha particle also called Helium-4:

$$^{10}B + n \rightarrow {}^{7}Li + {}^{4}He \quad . \qquad (3.4)$$

This reaction is used in some neutron detectors which are employed during the early stages of reactor start-up; the reason for this method of neutron detection is that the emitted alpha particle, being a doubly charged ion, is readily detected in an ionization chamber.

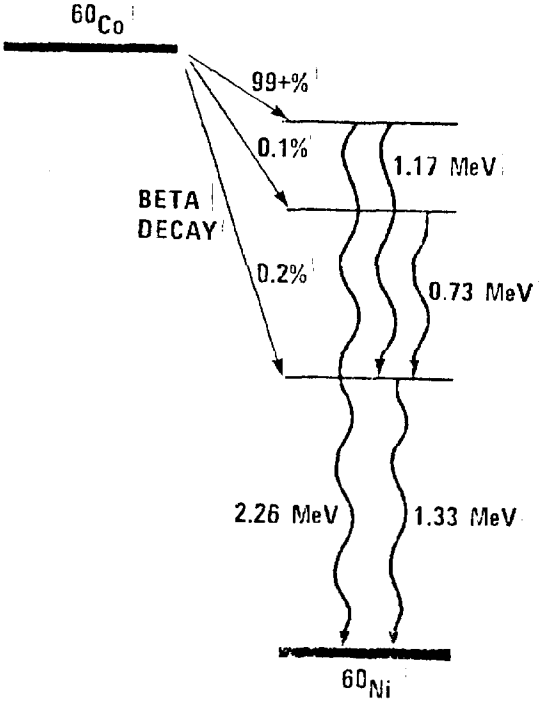

FIG. 3.1: Decay processes for Cobalt-60 and energy levels of Nickel-60.

The neutron-nucleus elastic scattering process is important because it represents the principle means whereby high energy neutrons lose energy and thereby enhance the neutron reproduction process. In a neutron-deuterium interaction we may represent this process simply by

$$D + n \rightarrow n + D \ . \tag{3.5}$$

It is informative to consider the neutron energy loss in such an elastic scattering process. Consider the head-on elastic scattering collision as suggested in Fig. 3.2.

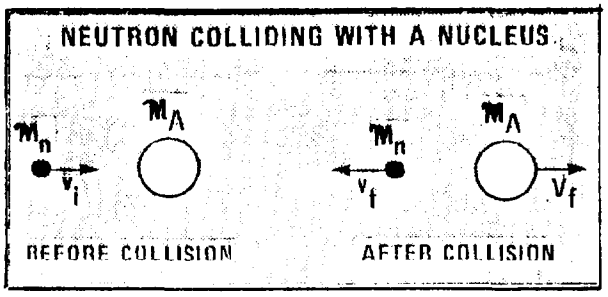

FIG. 3.2: Elastic collision between a thermal neutron and a stationary nucleus. The symbols m_n and m_A denote the mass of the neutron and nucleus respectively; the several particle speeds v_i, v_f and V_f correspond to the particle directions noted on this figure.

The subscripts i and f refer to the initial and final state of the variables; the other symbols are used as previously defined. By conservation of kinetic energy, we must have

$$\frac{1}{2} m_n v_i^2 = \frac{1}{2} m_n v_f^2 + \frac{1}{2} m_A V_f^2 \ , \tag{3.6}$$

where m_n and m_A represent the mass of the neutron and nucleus respectively, v_i and v_f represent the initial and final neutron speed; and V_f represents the final speed of the nucleus. By conservation of momentum, we also have

$$m_n v_i = m_A V_f - m_n v_f \ . \tag{3.7}$$

Equation (3.6) may be rewritten as

$$v_i^2 - v_f^2 = \frac{m_A}{m_n} V_f^2 \ , \tag{3.8}$$

or

$$(v_i - v_f)(v_i + v_f) = \frac{m_A}{m_n} V_f^2 \ , \tag{3.9}$$

while Eq. (3.7) reduces to

$$v_i + v_f = \frac{m_A}{m_n} V_f \ . \tag{3.10}$$

Dividing Eq. (3.1) by Eq. (3.10) provides the expression for the speed of the nucleus after collision, V_f,

$$V_f = v_i - v_f \ , \tag{3.11}$$

When this expression for V_f is substituted into Eq. (3.10) we obtain an explicit expression for the kinetic energy of the neutron after the head-on elastic scattering collision in terms of its initial kinetic energy:

$$v_i + v_f = \frac{m_A}{m_n} (v_i - v_f) \ . \tag{3.12}$$

That is

$$v_f(1 + \frac{m_A}{m_n}) = v_i(1 - \frac{m_A}{m_n}) \ , \tag{3.13}$$

or squaring each side and multiplying by 1/2 m_n, we obtain

$$E_f = [(1 - \frac{m_A}{m_n})/(1 + \frac{m_A}{m_n})]^2 E_i \ . \tag{3.14}$$

To a very close approximation we may use

$$A = \frac{m_A}{m_n} \ , \tag{3.15}$$

where A is the atomic mass number of the nucleus as previously used. We now define a parameter α,

$$\alpha = (\frac{1 - A}{1 + A})^2 \ , \tag{3.16}$$

and therefore write Eq. (3.14) compactly as

$$E_f = \alpha E_i \ . \tag{3.17}$$

This simple expression, relating the neutron energy before scattering with a nucleus, E_i, to its energy after collision, E_f, provides some useful insight to a fuller understanding of neutron slowing down by elastic scattering collisions. Clearly, the parameter α depends upon the atomic mass number of the nuclear specie with which the neutron undergoes an elastic scattering interaction. Consider a heavy nucleus such as, say Uranium-238. For this case, according to Eq. (3.16)

$$\alpha = (\frac{1 - 238}{1 + 238})^2 = 0.9833 \ . \tag{3.18}$$

By reference to Eq. (3.17) we conclude that a neutron will still retain 98.33% of its initial energy per elastic scattering interaction. In contrast, for a light nucleus, say deuterium, we have

$$\alpha = (\frac{1 - 2}{1 + 2})^2 = 0.1111 \ , \tag{3.19}$$

which means that the neutron will only retain 11.11% of its initial energy; hence 88.89% of the neutron energy is lost in one elastic scattering interaction with deuterium in the heavy water. Clearly, since each neutron interaction may lead a possible non-productive neutron removal by neutron absorption, a light nucleus is to be preferred for neutron slowing down purposes since this minimizes the number of neutron-nucleus interactions. This conclusion represents the physical basis for the long and continuing interest in light nuclei as moderators: light water, heavy water, graphite, and berillium.

Equation (3.17) describes the change in neutron energy specifically for the case of a head-on collision. A detailed analysis can be undertaken to show that for elastic scattering collisions the final energy is also related to the scattering angle of the neutron and can be shown to be given by

$$E_f = \frac{1}{2}[(1 + \alpha) + (1 - \alpha)\cos\theta_c]E_i \quad , \tag{3.20}$$

where θ_c represents the scattering angle in the centre of mass system. Note that if θ_c is 180°, a situation which applies for a head-on collision, the above equation reduces to Eq. (3.18).

3.2 FISSILE AND FERTILE NUCLEI

The most important and most dominant fission process in a CANDU nuclear reactor occurs in Uranium-235 in the Uranium-Oxide fuel pellets:

$$^{235}U + n \rightarrow (FP)_1 + (FP)_2 + \nu n \quad . \tag{3.21}$$

As indicated in Chapter 2, Section 4, the CANDU reactor does breed fissile Plutonium-239 by neutron capture in Uranium-238. The fission in Plutonium-239 is similarly represented by

$$^{239}Pu + n \rightarrow (FP)_1 + (FP)_2 + \nu n \quad . \tag{3.22}$$

There exists a definite, although small probability that, before Plutonium-239 can fission, it may capture the neutron and beoome transmuted to Plutonium-240. A further neutron capture leads to another readily fissile nucleus, Plutonium-241. The entire process may be represented by the following sequence of processes:

$$^{239}Pu + n \rightarrow {}^{240}Pu + n \rightarrow {}^{241}Pu \quad , \tag{3.23}$$

$$^{241}Pu + n \rightarrow (FP)_1 + (FP)_2 + \nu n \quad . \tag{3.24}$$

If some natural thorium should be contained within the nuclear fuel then, as we pointed out in Chapter 2, fissile Uranium-233 is produced which again fissions readily by thermal neutrons:

$$^{233}U + n \rightarrow (FP)_1 + (FP)_2 + \nu n \quad . \tag{3.25}$$

Up until now we have referred to Uranium-238 as being a nucleus which, as a consequence of capturing a neutron, initiates a nuclear transformation process yielding fissile Plutonium-239. It is known that, although Uranium-238 will not fission by thermal neutron, a high energy neutron can cause fission:

$$^{238}U + n \rightarrow (FP)_1 + (FP)_2 + \nu n \quad . \tag{3.26}$$

This is only possible if the neutron energy exceeds 1.3 MeV. Approximately two percent of the fission neutrons in a CANDU reactor emerge as a result of this fast fission process.

We summarize here the fissile and fertile nuclei in order of their importance in a CANDU reactor.

Fissile Nuclei: Uranium-235 (supplied in natural uranium fuel),
Plutonium-239 (by transmutation from Uranium-238 in natural uranium fuel),
Plutonium-241 (by transmutation from Plutonium-239),
Uranium-238 (associated with fast neutron fission only),
Uranium-233 (by transmutation if thorium present),

Fertile Nuclei: Uranium-238 (to produce Plutonium-239 by transmutation),
Thorium-232 (to produce Uranium-233 by transmutation).

3.3 FISSION NEUTRONS AND THERMAL NEUTRONS

The most energetic neutrons in a nuclear reactor are those which have just emerged from fission. These neutrons are not monoenergetic but follow a statistically reproducible distribution; in Fig. 3.3 we show the general shape of such a distribution. Although no satisfactory theory has been developed to describe this energy spectrum theoretically, a number of empirical functions have been obtained which provide a satisfactory fit to the experimental results. Two such distribution functions describing the number of fission neutrons per unit energy, $n_f(E)$, are given by

$$n_f(E) = 0.770e^{-0.775\sqrt{E}} \quad , \tag{3.27}$$

and

$$n_f(E) = 0.453e^{-1.034E}\sinh(2.29E) \quad , \tag{3.28}$$

where E is expressed in units of MeV.

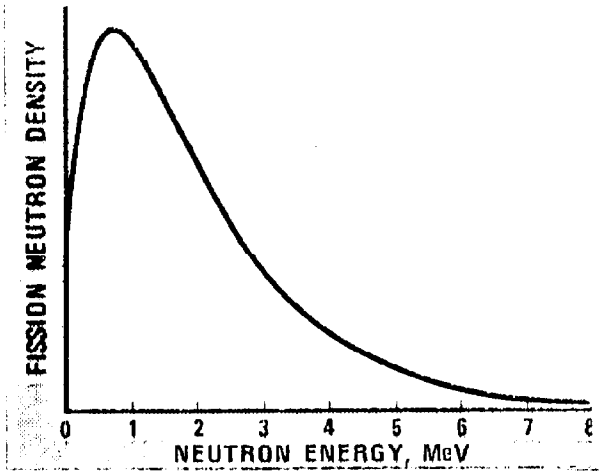

FIG. 3.3: Energy distribution of neutrons appearing in nuclear fission.

The most probable energy of a fission neutron is obtained by the condition

$$\frac{dn_f(E)}{dt} = 0 \quad , \tag{3.29}$$

where d/dE defines differentiation with respect to energy. In this manner the most probable energy of a neutron appearing as a result of fission can be found to have the value of 0.65 MeV. Of course, as soon as a neutron scatters from a nucleus its energy will decrease., Sec. 3.1.

The average fission neutron energy is defined by

$$\langle E \rangle = \int_0^\infty E\, n_f(E)dE \Big/ \int_0^\infty n_f(E)dE \quad . \tag{3.30}$$

When the above functions are inserted in Eq. (3.30) and the indicated integration carried out we obtain a value of about 1.8 MeV. The numerical values for the most probable energy and for the average energy may vary by some 10% for the several fissile nuclei.

While the fission neutrons possess the highest energies in a nuclear reactor - energies in the low MeV range - the lowest energy neutrons, called thermal neutrons, possess energies in the eV range. This lower energy range is associated with those neutrons which have attained a state of thermal equilibrium in the medium similar to that obtained by the molecules in a gas. Indeed, this analogy has been used to describe the energy distribution of neutrons and is called the Maxwell-Boltzmann distribution.

$$n_M(E) = \frac{2\pi n}{(\pi kT)^{3/2}} E^{1/2} \exp[-E/kT] \ , \tag{3.31}$$

where n is the total neutron density, k is the Boltzmann constant, T is the temperature on the Kelvin scale, and E is the energy of the neutrons.

This distribution is illustrated in Fig. 3.4; it is a very close approximation to the actual distribution of thermal neutrons in the moderator of the CANDU reactor. A slight distortion from this ideal representation generally exists and has the effect of shifting the distribution towards higher energies; this shift, called spectrum hardening, is attributed to preferential neutron absorption with decreasing energy and to neutron slowing down effects for higher energies.

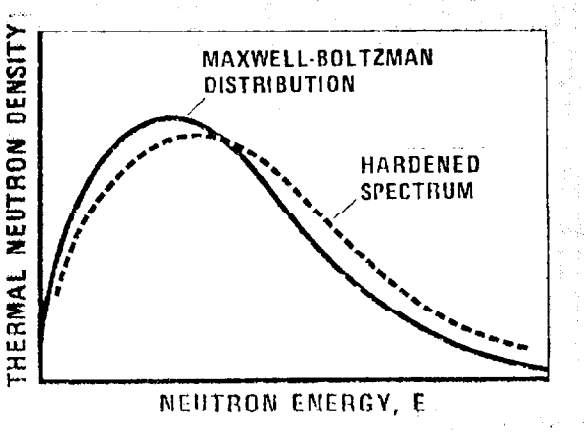

FIG. 3.4: Energy distribution of thermal neutrons in a reactor as predicted by theory (Maxwell-Boltzmann) and as observed.

The Maxwell-Boltzmann distribution is most useful in providing single-parameter descriptions of these thermal neutrons. For example, the most probable neutron energy is given by differentiation of Eq. (3.21):

$$\frac{dn_M(E)}{dE} = 0 \quad . \tag{3.32}$$

Solving for the energy which, by this definition, is the most probable neutron energy designated by E_p, results in

$$E_p = \frac{1}{2} kT \quad . \tag{3.33}$$

The most probable neutron speed is obtained by replacing E by its kinetic energy

$$\frac{1}{2} mv_p^2 = \frac{1}{2} kT \quad , \tag{3.34}$$

and solving for v_p

$$v_p = \frac{kT}{m} \quad . \tag{3.35}$$

For the case of room temperature we use T = 293°K to obtain

$$v_p = 2200 \text{ m/s} = 4920 \text{ mph.} \quad , \tag{3.36}$$

This corresponds to a neutron energy of

$$E_p = \frac{1}{2} m v_p^2 = 0.025 \text{ eV} \quad . \tag{3.37}$$

Many energy dependent neutron-nucleus interaction parameters are often quoted as the values corresponding to this energy of 0.025 eV (2200 m/s).

In Fig. 3.5 we provide a graphical representation of the relative location of the Maxwell-Boltzman distribution and the energy distribution of the fission neutrons. On this figure we indicate the three dominant energy ranges which are used to describe neutron processes in a nuclear reactor. The intermediate energy range is important because neutrons must slow down through this range before they can become effective in contributing to the neutron chain reaction.

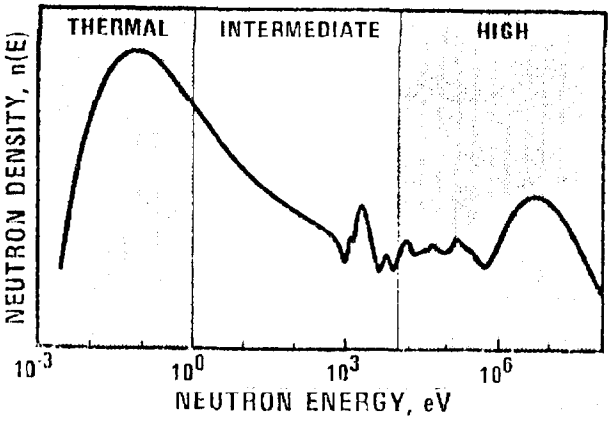

FIG. 3.5: Schematic representation of the distribution of neutrons with energy.

3.4 CROSS SECTIONS AND NEUTRON FLUX

We indicated in the preceding section that when neutrons first appear in the nuclear reactor they emerge as a consequence of a fission process and possess an energy in the million electron volt range. In their migration through the medium, these neutrons undergo scattering interactions which lead to a loss of kinetic energy until the neutron population attains a thermal equilibrium in the medium; this represents an energy loss of 8 orders of magnitude. Alternatively, these neutrons may be absorbed by a nuclide in the core and thus be lost; similarly, some neutrons may even escape from the reactor core and be captured in the reactor shield.

To provide a quantitative framework for our further analysis, we consider a space and energy dependent neutron density distribution function represented by $n(\underset{\rightarrow}{r},E)$ and defined as

$n(\underset{\rightarrow}{r},E)$ = number of neutrons per unit volume at the point $\underset{\rightarrow}{r}$ in the reactor and per unit energy at E.

The total number of neutrons per unit volume of all energies at the point $\underset{\rightarrow}{r}$ is hence given by integration over all neutron energies

$$n(\underset{\rightarrow}{r}) = \int_0^\infty n(\underset{\rightarrow}{r},E)dE \quad . \tag{3.38}$$

If we now suppose that the material of interest contains $N(\underset{\rightarrow}{r})$ nuclei per unit volume at each point in the medium, we may assert that the interaction density-rate per unit energy for the i'th type of interaction, designated by $F_i(\underset{\rightarrow}{r},E)$, may be represented by an equation which literally may be stated as

number of i'th type neutron-nucleus interactions per unit energy per unit volume per second, $F_i(\underset{\rightarrow}{r},E)$	proportional to	1) densities of nuclei, $N(\underset{\rightarrow}{r})$ 2) density of neutrons per unit energy, $n(\underset{\rightarrow}{r},E)$ 3) speed of neutrons of energy E, v(E).

Symbolically, we may write this proportionality statement as

$$F_i(\underset{\rightarrow}{r},E) \propto N(\underset{\rightarrow}{r})n(\underset{\rightarrow}{r},E)v(E) \quad , \tag{3.39}$$

or, introducing an energy dependent proportionality constant, $\sigma_i(E)$, we write

$$F_i(\underset{\rightarrow}{r},E) = \sigma_i(E)N(\underset{\rightarrow}{r})n(\underset{\rightarrow}{r},E)v(E) \quad . \tag{3.40}$$

Dimensionality considerations require that $\sigma_i(E)$ possess units of area, cm^2. The name microscopic cross section has been assigned to this parameter. It has been found convenient to define the unit barn, abbreviated b, by

$$1 \text{ b} = 10^{-24} cm^2 \quad . \tag{3.41}$$

The subscript i is used to specify the various possible neutron-nucleus interaction processes. For example, the following cases may be enumerated:

$\sigma_f(E)$: fission process,

$\sigma_a(E)$: absorption process,

$\sigma_c(E)$: capture process,

$\sigma_s(E)$: scattering process,

$\sigma_\gamma(E)$: capture-gamma process,

etc.

The total microscopic cross section is given by a summation of all individually identifiable processes

$$\sigma_t(E) = \sigma_a(E) + \sigma_s(E) + ... \qquad (3.42)$$

Here we have defined

$$\sigma_a(E) = \sigma_c(E) + \sigma_f(E) \ . \qquad (3.43)$$

Figure 3.6 provides a graphical representation of the energy dependence of some microscopic cross sections of interest to the CANDU system. It has been found that the microscopic total cross sections for many nuclei possess an energy dependence which may be characterized by the three dominant features as shown in Fig. 3.7

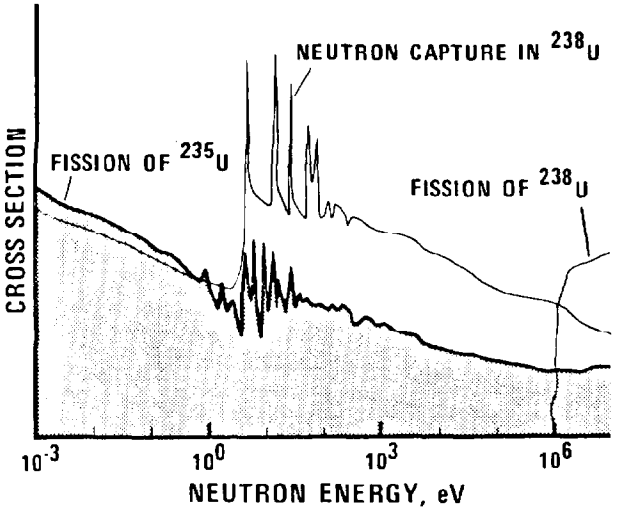

FIG. 3.6: Energy dependence of the fission and capture cross section in uranium.

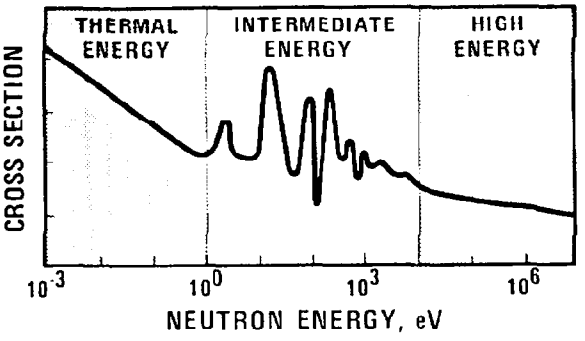

FIG. 3.7: Typical energy dependence of neutron capture cross sections for many nuclei.

The extensive use of the products $\sigma_i(E)N(\underset{\rightarrow}{r})$ and $n(\underset{\rightarrow}{r},E)v(E)$ has led to the widespread adoption of the following quantities. A macroscopic cross section $\Sigma_i(r,E)$ for the i'th type of neutron-nucleus process is defined by the product of the microscopic cross section and the atomic density of the interacting nuclei

$$\Sigma_i(\underset{\rightarrow}{r},E) = \sigma_i(E)N(\underset{\rightarrow}{r}) \ , \qquad (3.44)$$

and possesses units of cm^{-1}. The energy dependent neutron flux, $\phi(\underset{\rightarrow}{r},E)$, is defined by

$$\phi(\underset{\rightarrow}{r},E) = n(\underset{\rightarrow}{r},E)v(E) \ . \qquad (3.45)$$

The interaction density rate per unit energy for the i'th type of process is thus given by

$$F_i(\underset{\rightarrow}{r},E) = \Sigma_i(\underset{\rightarrow}{r},E)\phi(\underset{\rightarrow}{r},E) \ , \tag{3.46}$$

or, if no misunderstanding can arise, simply as

$$F_i = \Sigma_i\phi \ . \tag{3.47}$$

As an example of the use of these various terms we write the total number of fission reactions per unit volume at $\underset{\rightarrow}{r}$ in the reactor core as

$$F_f(\underset{\rightarrow}{r}) = \int_0^\infty \Sigma_f(\underset{\rightarrow}{r},E)\phi(\underset{\rightarrow}{r},E)dE \ . \tag{3.48}$$

Similarly, the total number of fission neutrons appearing per cm^3 at $\underset{\rightarrow}{r}$ is given by

$$S_f(\underset{\rightarrow}{r}) = \int_0^\infty \nu(E)\Sigma_f(\underset{\rightarrow}{r},E)\phi(\underset{\rightarrow}{r},E)dE \ , \tag{3.49}$$

where $\nu(E)$ represents the average number of fission neutrons appearing per unit energy per fission as a function of energy.

It is now a simple matter to derive numerous expressions of interest in reactor analysis. For example, the power density at a point of interest is obtained directly from the fission density rate by the use of the appropriate conversion constants which relate the power density in MeV/s to the desired power units of interest. Thus, we write

$$p(\underset{\rightarrow}{r}) = K\int_0^\infty \Sigma_f(\underset{\rightarrow}{r},E)\phi(\underset{\rightarrow}{r},E)dE \ , \tag{3.50}$$

for the power density and, by an integration over the reactor volume, V, we get

$$P = K\int_0^\infty\int_V \Sigma_f(\underset{\rightarrow}{r},E)\phi(\underset{\rightarrow}{r},E)d\underset{\rightarrow}{r}dE \ , \tag{3.51}$$

for the total reactor power. Here K is the appropriate conversion constant.

3.5 NEUTRON SLOWING DOWN

Neutrons possessing energies above 1 eV and below 10 keV are commonly called intermediate-energy neutrons. All fission neutrons must, if they are to contribute effectively to the nuclear chain reaction, pass through this intermediate energy domain. The dominant mechanism for slowing down to thermal energies is

the elastic scattering process discussed in Section 3.2. We recall that if a neutron with an initial energy E_i is elastically scattered by a nucleus A_X through an angle θ_c in the centre of mass coordinate system, then its final energy will be given by E_f:

$$E_f = \frac{1}{2}[(1 + \alpha) + (1 - \alpha)\cos\theta_c]E_i \ , \tag{3.52}$$

where α is a function of the atomic mass number of the nucleus and defined by

$$\alpha = \left(\frac{A - 1}{A + 1}\right)^2 \ . \tag{3.53}$$

A minimum neutron energy can be attained by a direct head-on collision. This energy E_{Min} is given by

$$E_{Min} = \alpha E_i \ . \tag{3.54}$$

The final neutron energy therefore is in the energy interval

$$\alpha E_i \leq E_f \leq E_i \ . \tag{3.55}$$

Whenever a neutron of energy E_i enters into a reaction with a nucleus, the probability that a scattering event will take place is given by the appropriate ratio of cross sections evaluated at energy E_i:

$$\text{Probability of neutron scattering at energy } E_i = \frac{\sigma_s(E_i)}{\sigma_t(E_i)} \ . \tag{3.56}$$

The probability of neutron capture is similarly given by

$$\text{Probability of neutron capture at energy } E_i = \frac{\sigma_c(E_i)}{\sigma_t(E_i)} \ . \tag{3.57}$$

It has been found that for many elements this ratio, Eq. (3.57), can be undesirably high; this implies that many neutrons become captured. The effect of this parasitic capture in the intermediate energy domain can be minimized by a choice of moderators which minimize the number of interactions in the slowing down process. As indicated in Chapter 2, this would suggest that it would be desirable to minimize α. These two characteristics suggest that a good neutron moderator possess the following two properties:

1) low atomic mass number to maximize the energy loss of neutrons per elastic scattering, and
2) a large scattering cross section or, equivalently, very small capture cross section.

We will consider quantifying these terms.

Consider a fission neutron of energy E_{FISS} which undergoes an elastic scattering collision and has its energy reduced to E_1. This neutron then undergoes another

scattering process to attain energy E_2. This process continues through a total of N collisions until thermal energy E_{THER} is attained. We write the logarithm of the ratio E_{FISS}/E_{THER} as

$$\ell n\left(\frac{E_{FISS}}{E_{THER}}\right) = \ell n\left(\frac{E_{FISS}}{E_1} \times \frac{E_1}{E_2} \times \ldots \times \frac{E_i}{E_f} \times \ldots \times \frac{E_{N-1}}{E_{THER}}\right) ,$$

$$= \ell n\left(\frac{E_{FISS}}{E_1}\right) \times \ldots \times \ell n\left(\frac{E_i}{E_f}\right) \times \ldots \times \ell n\left(\frac{E_{N-1}}{E_{THER}}\right) . \qquad (3.58)$$

We thus have a sum of N terms each representing a logarithmic energy decrement per elastic scattering collision. This term has been studied extensively and, under rather general conditions, can be shown to be given in terms of the atomic mass number of the moderator by

$$\ell n\left(\frac{E_i}{E_f}\right) = 1 - \frac{(A-1)^2}{2A} \ell n\left(\frac{A+1}{A-1}\right) . \qquad (3.59)$$

For $A > 5$, this expression can be approximated by

$$\ell n\left(\frac{E_i}{E_f}\right) \simeq \frac{2}{A + 2/3} . \qquad (3.60)$$

It has become customary to use the symbol ξ for the logarithmic energy decrement. That is

$$\xi = \ell n\left(\frac{E_i}{E_f}\right) . \qquad (3.61)$$

We may now identify a parameter which can be used to compare the effectiveness of a moderator in slowing down neutrons. In view of our discussion of the importance of a large scattering cross section, the desirability of a small absorption cross section, and the necessity of a large neutron energy loss per elastic scattering collision, we define the term moderating ratio by

$$\text{Moderating Ratio} = \frac{\sigma_s}{\sigma_a} \xi . \qquad (3.62)$$

Typical values appropriate to several possible moderators are listed in Table 3.1 where we note the excellent neutron moderating property of heavy water.

Moderator	Moderating Ratio
Heavy water (pure)	4900
Heavy water (0.25% H_2O)	2100
Graphite	170
Berillium	150
Light Water	70

TABLE 3.1: Moderating ratios for several moderators.

We can extract an additional useful parameter from this type of an analysis. Referring to Eq. (3.58) and noting that since N_c collisions are represented - each of which is associated with a logarithmic energy decrement - we write

$$\ell n\left(\frac{E_{FISS}}{E_{THER}}\right) = N_c \xi \ . \tag{3.63}$$

Solving for N_c using $E_{FISS} = 2 \times 10^6$eV and $E_{THER} = 0.025$ eV yields

$$N_c = \frac{\ell n(8 \times 10^7)}{\xi} = \frac{10.8}{\xi} \ , \tag{3.64}$$

where ξ is given as a function of atomic mass number by Eq. (3.59). We may use this expression to determine the number of collisions a neutron encounters from an initial fission energy of 2 MeV to a thermal energy of 0.025 eV if its logarithmic energy decrement is given by ξ. Typical values for a number of materials are listed in Table 3.2.

Material	Number of Elastic Collisions
Hydrogen-1	85
Carbon-2	110
Iron-56	516
Uranium-238	>2100

TABLE 3.2: Number of elastic neutron collisions required to slow down from fission to thermal energies.

3.6 ENERGY AVERAGING

In the preceding sections we have indicated the importance of neutron energy to a more complete description of neutron induced processes. Although these expressions involving integration over energy are correct, it is obvious that the inclusion of the energy of the neutrons represents a considerable complication in many reactor calculations. As a consequence, methods of establishing energy-averaged or energy-equivalent expressions have been sought to permit a more practical means of calculating reactor parameters. We will consider some of these procedures and then identify the most frequently used expressions.

We have previously indicated that interaction densities, such as, say the absorption density rate, could be written as a product of macroscopic cross section and the neutron flux. That is

$$F_a \sim \Sigma_a \phi \ , \tag{3.65}$$

where we recognize that both the macroscopic cross section Σ_a and the flux ϕ are, in general, space dependent and energy dependent. If we restrict ourselves to a localized point description we may eliminate the spatial dependence and concentrate on the energy dependence. The exact neutron absorption density rate is clearly given by

$$F_a = \int_0^\infty \Sigma_a(E)\phi(E)dE = N\int_0^\infty \sigma_a(E)n(E)\sqrt{\frac{2E}{m}}\,dE \ , \tag{3.66}$$

where we have used the definitions

$$\Sigma_a(E) = \sigma_a(E)N \tag{3.67}$$

and

$$\phi(E) = n(E)v = n(E)\sqrt{\frac{2E}{m}} \ . \tag{3.68}$$

Since $\sigma_a(E)$ is generally known from experiment, the remaining problem then is to determine the energy dependent neutron density. As we discussed previously, Chapter 3, Section 3.3, the CANDU reactor is characterized by a highly thermalized neutron population which means that n(E) can be written as

$$n(E) = \frac{2\pi n}{(\pi kT)^{3/2}} E^{1/2}\exp[-E/kT] \ . \tag{3.69}$$

If, in addition, $\sigma_a(E)$ varies inversely with neutron speed then the integration can be carried out immediately to yield

$$F_a = \Sigma_a(E_o)nv_o = \Sigma_a(E_o)\phi_o \ , \tag{3.70}$$

where v_o is equal to 2200 m/s, that is the speed corresponding to peak of the Maxwellian distribution; $\Sigma_a(E_o)$ is the macroscopic cross section corresponding to the energy E_o = 0.025 eV.

If the absorption cross section does not vary inversely with speed then a correction factor has to be included. These factors are called g-factors and are dependent upon the medium temperature. In general, we have for absorption

$$F_a = g_a(T)\Sigma_a(E_o)\phi_o \ , \tag{3.71}$$

while for fission we write

$$F_f = g_f(T)\Sigma_f(E_o)\phi_o \ . \tag{3.72}$$

Typical g-factors are listed in Table 3.3.

T(°C)	^{235}U		^{239}Pu	
	$g_a(T)$	$g_f(T)$	$g_a(T)$	$g_f(T)$
200	0.9973	1.0025	1.3388	1.2528
400	1.0010	1.0068	1.8905	1.6904
600	1.0072	1.0128	2.5321	2.2037
800	1.0146	1.0201	3.1006	2.6595
1000	1.0226	1.0284	3.5353	3.0079

TABLE 3.3: Selected g-factors for Uranium-235 and Plutonium-239 as a function of temperature.

In addition to the 2200 m/s neutron flux, ϕ_o, and the 0.025 eV macroscopic cross section, $\Sigma(E_o)$, there are other energy-averaged definitions which are frequently used. For example, the thermal flux ϕ_T is defined by

$$\phi_T = \int_0^\infty \phi_M(E)dE = \int_0^\infty n_M(E)\sqrt{\frac{2E}{m}}\, dE \ . \tag{3.73}$$

Substituting the Maxwellian neutron distribution, Eq. (3.69), yields

$$\phi_T = \frac{2}{\sqrt{\pi}} n\left(\frac{2kT}{m}\right)^{1/2} \ , \tag{3.74}$$

which can also be written as

$$\phi_T = \frac{2}{\sqrt{\pi}} n v_T \ . \tag{3.75}$$

This expression, defining the thermal flux is clearly temperature dependent.

Referring to Eq. (3.71) and Eq. (3.72) together with the definitions for the flux, Eq. (3.70) and Eq. (3.74) suggest that it is still possible to write an interaction density as a product of the macroscopic cross section Σ and the neutron flux ϕ if the g-factors are included in the definition of Σ. For the i'th type of neutron-nucleus interaction density at a point $\underset{\sim}{r}$ in the reactor core, one may write

$$F_i(\underset{\sim}{r}) = \Sigma_i \phi(\underset{\sim}{r}) \ , \tag{3.76}$$

where it is understood that

$$\Sigma_i = g_i(T)\Sigma_i(E_o) \ , \tag{3.77}$$

and

$$\phi(\underset{\rightarrow}{r}) = n(\underset{\rightarrow}{r})v_0 \ . \tag{3.78}$$

We will use this notation in a subsequent analysis.